RANDIC INDEX AND EDGE ECCENTRIC CONNECTIVITY INDEX OF CERTAIN SPECIAL MOLECULAR GRAPHS

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ABSTRACT
In this paper, we determine the Randic index and edge eccentric connectivity index of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their r-corona molecular graphs.

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Keywords: Chemical graph theory, Randic index, Edge eccentric connectivity index, Fan molecular graph, Wheel molecular graph, Gear fan molecular graph, Gear wheel molecular graph, r-corona molecular graph.

1. INTRODUCTION
Wiener index, edge Wiener index, Hyper-wiener index, Randic index and edge eccentric connectivity index are introduced to reflect certain structural features of organic molecules. Several papers contributed to determine the Wiener index or Hyper-wiener index of special molecular graphs (See Yan, et al. [1], Gao and Shi [2] and Yan, et al. [3] for more detail). Let $P_n$ and $C_n$ be path and cycle with $n$ vertices. The molecular graph $F_n=\{v\} \vee P_n$ is called a fan molecular graph and the molecular graph $W_n=\{v\} \vee C_n$ is called a wheel molecular graph. Molecular graph $I_r(G)$ is called $r$-crown molecular graph of $G$ which splicing $r$ hang edges for every vertex in $G$. By adding one vertex in every two adjacent vertices of the fan path $P_n$, of fan molecular graph $F_n$, the resulting molecular graph is a subdivision molecular graph called gear fan molecular graph, denote as $\hat{F}_n$.

By adding one vertex in every two adjacent vertices of the wheel cycle $C_n$, of wheel molecular graph $W_n$. The resulting molecular graph is a subdivision molecular graph, called gear wheel molecular graph, denoted as $\hat{W}_n$. 
In 1975, Randic [4] introduced the Randic index as the sum of $d(u)d(v)^{-1/2}$ over all edges $uv$ of a molecular graph $G = (V, E)$, i.e.

$$R(G) = \sum_{uv \in E(G)} (d(u)d(v))^{-1/2}$$

where $d(u)$ denotes the degree of $u \in V(G)$. Later in 1998, Bollobas and Erdos [5] generalized this index by replacing $-1/2$ with any real number $\alpha$, which is called the general Randic index, i.e.

$$R_{\alpha}(G) = \sum_{uv \in E(G)} (d(u)d(v))^\alpha$$


Let $f = uv$ be an edge in $E(G)$. Then the degree of the edge $f$ is defined to be $\deg_G(u) + \deg_G(v) - 2$. For two edges $f_1 = u_1v_1, f_2 = u_2v_2$ in $E(G)$, the distance between $f_1$ and $f_2$, denoted by $d_G(f_1, f_2)$, is defined to be

$$d_G(f_1, f_2) = \min\{d_G(u_1, u_2), d_G(u_1, v_2), d_G(v_1, u_2), d_G(v_1, v_2)\}.$$ 

The eccentricity of an edge $f$, denoted by $ec_G(f)$, is defined as

$$ec_G(f) = \max\{d_G(f, e) \mid e \in E(G)\}.$$ 

The edge eccentric connectivity index of $G$, denoted by $\xi_c^e(G)$, is defined as

$$\xi_c^e(G) = \sum_{f \in E(G)} \deg_G(f)ec_G(f).$$ 

Xu and Guo [13] obtained various upper and lower bounds for this index of connected molecular graphs in terms of order, size, girth and the first Zagreb index of $G$, respectively. Other results on edge eccentric connectivity index can refer to Odabaş [14].

In this paper, we present the Randic index of $I_r(F_n), I_r(W_n), I_r(\bar{F}_n)$ and $I_r(\bar{W}_n)$. Also, the edge eccentric connectivity index of $I_r(F_n), I_r(W_n), I_r(\bar{F}_n)$ and $I_r(\bar{W}_n)$ are derived.
2. RANDIC INDEX

Theorem 1. 

\[ R(I_r(F_n)) = \frac{r}{\sqrt{n+r}} + \frac{2}{\sqrt{(n+r)(2+r)}} + \frac{n-2}{\sqrt{(n+r)(3+r)}} + \] 

\[ + \frac{2}{\sqrt{(2+r)(3+r)}} + \frac{n-3}{\sqrt{(3+r)(3+r)}} + \frac{2r}{\sqrt{2+r}} + \frac{(n-2)r}{\sqrt{3+r}}. \]

Proof. Let \( P_n = v_1v_2...v_n \) and the \( r \) hanging vertices of \( v_i \) be \( v_1^i, v_2^i, ..., v_r^i \) (\( 1 \leq i \leq n \)). Let \( v \) be a vertex in \( F_n \) beside \( P_n \), and the \( r \) hanging vertices of \( v \) be \( v_1, v_2, ..., v_r \). By the definition of Randic index, we have

\[ R(I_r(F_n)) = \sum_{i=1}^{r} (d(v)d(v^i))^{1/2} + \sum_{i=1}^{n} (d(v)d(v_i))^{1/2} + \sum_{i=1}^{n-1} (d(v_i)d(v_{i+1}))^{1/2} + \] 

\[ + \sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i)d(v_i^j))^{1/2} \]

\[ = \frac{r}{\sqrt{n+r}} + \left( \frac{2}{\sqrt{(n+r)(2+r)}} + \frac{n-2}{\sqrt{(n+r)(3+r)}} \right) + \left( \frac{2}{\sqrt{(2+r)(3+r)}} + \frac{n-3}{\sqrt{(3+r)(3+r)}} \right) \]

\[ + \left( \frac{2r}{\sqrt{2+r}} + \frac{(n-2)r}{\sqrt{3+r}} \right). \]

Corollary 1. 

\[ R(F_n) = \sqrt{\frac{2}{n}} + \frac{n-2}{\sqrt{3n}} + \frac{2}{\sqrt{6}} + \frac{n-3}{3}. \]

Theorem 2. 

\[ R(I_r(W_n)) = \frac{r}{\sqrt{n+r}} + \frac{n}{\sqrt{(n+r)(3+r)}} + \frac{n}{\sqrt{(3+r)(3+r)}} + \frac{nr}{\sqrt{3+r}}. \]

Proof. Let \( C_n = v_1v_2...v_n \) and \( v_1^i, v_2^i, ..., v_r^i \) be the \( r \) hanging vertices of \( v_i \) (\( 1 \leq i \leq n \)). Let \( v \) be a vertex in \( W_n \) beside \( C_n \), and \( v_1, v_2, ..., v_r \) be the \( r \) hanging vertices of \( v \). By the definition of Randic index, we have

\[ R(I_r(W_n)) = \sum_{i=1}^{r} (d(v)d(v^i))^{1/2} + \sum_{i=1}^{n} (d(v)d(v_i))^{1/2} + \sum_{i=1}^{n} (d(v_i)d(v_{i+1}))^{1/2} + \] 

\[ + \sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i)d(v_i^j))^{1/2} \]

\[ = \frac{r}{\sqrt{n+r}} + \frac{n}{\sqrt{(n+r)(3+r)}} + \frac{n}{\sqrt{(3+r)(3+r)}} + \frac{nr}{\sqrt{3+r}}. \]
Corollary 2. $R(W_n) = \sqrt{\frac{n}{3} + \frac{n}{3}}$.

Theorem 3. $R(I_r(\tilde{F}_n)) = \frac{r}{\sqrt{n + r}} + \frac{2}{\sqrt{(n + r)(2 + r)}} + \frac{n - 2}{\sqrt{(n + r)(3 + r)}} + \frac{(n + 1)r}{\sqrt{2 + r}} + \frac{(n - 2)r}{\sqrt{3 + r}} + \frac{2}{\sqrt{(2 + r)(2 + r)}} + \frac{2n - 4}{\sqrt{(2 + r)(3 + r)}}$.

Proof. Let $P_i = v_1v_2\ldots v_n$ and $v_{i,i+1}$ be the adding vertex between $v_i$ and $v_{i+1}$. Let $v_i^1, v_i^2, \ldots, v_i^r$ be the $r$ hanging vertices of $v_i (1 \leq i \leq n)$. Let $v_{i,i+1}^1, v_{i,i+1}^2, \ldots, v_{i,i+1}^r$ be the $r$ hanging vertices of $v_{i,i+1} (1 \leq i \leq n - 1)$. Let $v$ be a vertex in $F_n$ beside $P_n$, and the $r$ hanging vertices of $v$ be $v^1, v^2, \ldots, v^r$. By virtue of the definition of Randic index, we deduce

$$R(I_r(\tilde{F}_n)) = \sum_{i=1}^{r} (d(v) d(v_i^1))^{1/2} + \sum_{i=1}^{n} (d(v) d(v_i^2))^{1/2} + \sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i^1) d(v_j^1))^{1/2} + \sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i^2) d(v_j^2))^{1/2}$$

$$= \frac{r}{\sqrt{n + r}} + \frac{2}{\sqrt{(n + r)(2 + r)}} + \frac{n - 2}{\sqrt{(n + r)(3 + r)}} + \frac{2r}{\sqrt{2 + r}} + \frac{(n - 2)r}{\sqrt{3 + r}} + \frac{2n - 4}{\sqrt{(2 + r)(3 + r)}}.$$

Corollary 3. $R(\tilde{F}_n) = \sqrt{\frac{2}{n} + \frac{n - 2}{\sqrt{3n}} + 1 + \frac{2(n - 2)}{\sqrt{6}}}$.

Theorem 4. $R(I_r(\tilde{W}_n)) = \frac{r}{\sqrt{n + r}} + \frac{n}{\sqrt{(n + r)(3 + r)}} + \frac{nr}{\sqrt{(3 + r)(2 + r)}} + \frac{2n}{\sqrt{2 + r}} + \frac{nr}{\sqrt{(3 + r)(2 + r)}}$.

Proof. Let $C_n = v_1v_2\ldots v_n$ and $v$ be a vertex in $W_n$ beside $C_n$. $v_{i,i+1}$ be the adding vertex between $v_i$ and $v_{i+1}$. Let $v^1, v^2, \ldots, v^r$ be the $r$ hanging vertices of $v$ and $v_i^1, v_i^2, \ldots, v_i^r$ be the $r$ hanging vertices of $v_i (1 \leq i \leq n)$. Let $v_{n,n+1} = v_1, v_{n+1,n+1} = v_i^1, v_{i+1,n+1} = v_i^2, \ldots, v_{n+1,n+1} = v_i^r$ be the $r$ hanging vertices of $v_{i,i+1} (1 \leq i \leq n)$. In view of the definition of Randic index, we deduce
\[ R(I_r(\tilde{W}_n)) = \sum_{i=1}^{r} (d(v)d(v'))^{-1/2} + \sum_{i=1}^{n} (d(v)d(v_i))^{-1/2} + \sum_{j=1}^{r} \sum_{i=1}^{n} (d(v_j)d(v'_i))^{-1/2} + \]

\[ \sum_{i=1}^{n} (d(v_i)d(v_{i+1}))^{-1/2} + \sum_{i=1}^{r} (d(v_{i+1})d(v'_{i+1}))^{-1/2} + \sum_{j=1}^{r} \sum_{i=1}^{n} (d(v_{i+1})d(v'_{i+1}))^{-1/2} \]

\[ = \frac{r}{\sqrt{n+r}} + \frac{n}{\sqrt{(n+r)(3+r)}} + \frac{nr}{\sqrt{3+r}} + \frac{n}{\sqrt{(3+r)(2+r)}} + \frac{n}{\sqrt{2+r}}. \]

**Corollary 4.** \[ R(\tilde{W}_n) = \frac{n}{\sqrt{3}} + \frac{2n}{\sqrt{6}}. \]

## 3. Edge Eccentric Connectivity Index

**Theorem 5.** \[ \xi^e_r(I_r(F_n)) = n^2 + n(2r^2 + 11r + 9) + r^2 - 9r - 14. \]

**Proof.** By the definition of edge eccentric connectivity index, we have

\[ \xi^e_r(I_r(F_n)) = \sum_{i=1}^{r} \deg_G(vv')ec_G(vv') + \sum_{i=1}^{n} \deg_G(vv_i)ec_G(vv_i) + \sum_{i=1}^{n} \deg_G(vv_{i+1})ec_G(vv_{i+1}) \]

\[ + \sum_{i=1}^{n} \sum_{j=1}^{r} \deg_G(v_i,v'_i)ec_G(v_i,v'_i) \]

\[ = r(n+r+1) + (2(n+2r)+(n-2)(n+2r+1)) + (2\times2(2r+3)+(n-3)\times2(2r+4)) + \]

\[ r(2\times2(r+1)+(n-2)\times2(r+2)) \]

\[ = n^2 + n(2r^2 + 11r + 9) + r^2 - 9r - 14. \]

**Corollary 5.** \[ \xi^e_r(F_n) = n^2 + 9n - 14. \]

**Theorem 6.** \[ \xi^e_r(I_r(W_n)) = n^2 + n(2r^2 + 11r + 9) + r^2 - 9. \]

**Proof.** By the definition of edge eccentric connectivity index, we have

\[ \xi^e_r(I_r(W_n)) = \sum_{i=1}^{r} \deg_G(vv')ec_G(vv') + \sum_{i=1}^{n} \deg_G(vv_i)ec_G(vv_i) + \]

\[ \sum_{i=1}^{n} \deg_G(vv_{i+1})ec_G(vv_{i+1}) + \sum_{i=1}^{n} \sum_{j=1}^{r} \deg_G(v_i,v'_i)ec_G(v_i,v'_i) + \]

\[ = r(n+r-1) + n(n+2r+1) + 2n(2r+4) + 2nr(r+2) \]

\[ = n^2 + n(2r^2 + 11r + 9) + r^2 - 9. \]
Corollary 6. $\mathcal{E}_e^c(W_n) = n^2 + 9n$.

Theorem 7. $\mathcal{E}_e^c(I_r(\tilde{F}_n)) = 2n^2 + n(7r^2 + 28r + 20) - 2r^2 - 24r - 28$.

Proof. By virtue of the definition of edge eccentric connectivity index, we get

$$\mathcal{E}_e^c(I_r(\tilde{F}_n)) = \sum_{i=1}^{r} \deg_G(v^{i'})ec_G(v^{i'}) + \sum_{i=1}^{n} \deg_G(vv_i)ec_G(vv_i) +$$

$$\sum_{i=1}^{n} \sum_{j=1}^{r} \deg_G(v^{i'}v^{j'})ec_G(v^{i'}v^{j'}) + \sum_{i=1}^{n-1} \deg_G(vv_{i+1})ec_G(vv_{i+1}) +$$

$$\sum_{i=1}^{n} \deg_G(v_{i,i+1}v^{i+1})ec_G(v_{i,i+1}v^{i+1}) + \sum_{i=1}^{n} \deg_G(v_{i,i+1}v^{j'})ec_G(v_{i,i+1}v^{j'})$$

$$= 2r(n + r - 1) + (2 \times 2(n + 2r) + (n - 2) \times 2(n + 2r + 1)) +$$

$$r(2 \times 3(r + 1) + (n - 2) \times 3(r + 2)) + (3(2r + 2) + (n - 2) \times 3(2r + 3)) +$$

$$(3(2r + 2) + (n - 2) \times 3(2r + 3)) + (n - 1)4r(r + 1)$$

$$= 2n^2 + n(7r^2 + 28r + 20) - 2r^2 - 24r - 28.$$ 

Corollary 7. $\mathcal{E}_e^c(\tilde{F}_n) = n^2 + 13n - 18$.

Theorem 8. $\mathcal{E}_e^c(I_r(\tilde{W}_n)) = 2n^2 + n(7r^2 + 28r + 20) + 2r^2 - 2r$.

Proof. In view of the definition of edge eccentric connectivity index, we deduce

$$\mathcal{E}_e^c(I_r(\tilde{W}_n)) = \sum_{i=1}^{r} \deg_G(v^{i'})ec_G(v^{i'}) + \sum_{i=1}^{n} \deg_G(vv_i)ec_G(vv_i) +$$

$$\sum_{i=1}^{n} \sum_{j=1}^{r} \deg_G(v^{i'}v^{j'})ec_G(v^{i'}v^{j'}) + \sum_{i=1}^{n} \deg_G(vv_{i+1})ec_G(vv_{i+1}) +$$

$$\sum_{i=1}^{n} \deg_G(v_{i,i+1}v^{i+1})ec_G(v_{i,i+1}v^{i+1}) + \sum_{i=1}^{n} \deg_G(v_{i,i+1}v^{j'})ec_G(v_{i,i+1}v^{j'})$$

$$= 2r(n + r - 1) + 2n(n + 2r + 1) + 3nr(r + 2) + 3n(2r + 3) + 3n(2r + 3) + 4nr(r + 1)$$

$$= 2n^2 + n(7r^2 + 28r + 20) + 2r^2 - 2r.$$ 

Corollary 8. $\mathcal{E}_e^c(\tilde{W}_n) = n^2 + 13n$.

4. CONCLUSION AND DISCUSSION

In this paper, we determine the Randic index and edge eccentric connectivity index of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph,
and their $r$-corona molecular graphs. The Hosoya index $Z(G)$ of molecular graph $G$ is defined as the number of subsets of the edge set $E(G)$ in which no two edges are adjacent in $G$, i.e., $Z(G)$ is the total number of matchings of $G$. Hence, the Hosoya index of a molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their $r$-corona molecular graphs should considered as our further work.

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